MODELING THE DISTRIBUTION OF A "MEME" IN A SIMPLE AGE DISTRIBUTION POPULATION: I. A KINETICS APPROACH AND SOME ALTERNATIVE MODELS

Matthew Witten

Center for High Performance Computing
University of Texas, Austin, TX 78758-4497

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Abstract. Although there is a growing historical body of literature relating to the mathematical modeling of social and historical processes, little effort has been placed upon modeling the spread of an idea element "meme" in such a population. In this paper we review some of the literature and we then consider a simple kinetics approach, drawn from demography, to model the distribution of a hypothetical "meme" in a population consisting of three major age groups. KEYWORDS: Meme, idea, age-structure, compartment, sociobiology, kinetics model.

I. Introduction.

Mathematical approaches to culture transformation, social evolution, and history have only recently come to the fore. We begin with a brief overview of the historical literature on the mathematical modeling of culture changes.

Perhaps the first paper appearing in this area is a paper by Rashevsky (1939). This paper discusses various mathematical approaches towards a theory of human relations. Between 1939 and 1968 Rashevsky published some twenty-one papers on the mathematical modeling of social dynamics and history; covering such topics as mass behavior, aggressiveness, imitative behavior, a mathematical approach to history, mathematical biology of social behavior, and the topology of life. An excellent bibliography and reference set on the work of Rashevsky may be found in Rashevsky (1968) and (1972). Another interesting text which looks for the same type of unity between the physical, social, and biological sciences is Stone (1966). Stone's work provides a number of examples of how mathematics may be used to provide a formal description of systems in such fields as demography, anthropology, sociology, and economics.

During this period of time, 1939-1972, another major attempt was made to formalize human behavior. Based upon korzybakian premises Hilgartner (1965) constructs a theory of psychodynamics of human behaviors. The argument of these papers, which is formalized in Hilgartner and Randolph (1969 a,b), is that human behavior shows a *postulational structure*. That is, any human act may be analyzed as if it were a *logical system*, proceeding from some set of *assumptions* which in turn make use of some *undefined terms*, and utilizing some *grammar* or *modus operandi*. Based upon this construction, the role of *conclusion* or *theorem* is filled by the action in question. Unfortunately, this series of papers is never extended to any real world application.

Perhaps the next major inroad, in attempting to rethink the application of mathematical techniques to the study of human societies, is the work of Renfrew and Cooke (1979). This collection of essays contains an amazing diversity of applications of mathematics to questions of culture change. Examples, in this reference, extend from prehistoric society to more recent social structures.

1979 also marked the appearance of E.O. Wilson's classic Sociobiology (Wilson (1979)) which argued that nearly all behavior is seen to support and be produced by competitive genetic interest. According to the sociobiology theory, each animal acts to promote the propagation of its own genes, competing ruthlessly not only with other species but with members of its own for scarce but essential resources. This competition is not necessarily a conscious one, and it has the goal of leaving the greatest possible number of its own offspring to breed future generations. The animal will *occasionally* behave altruistically towards its own kin due to the genes they carry in common. However, otherwise it is full competition.

In a further attempt to understand the relationship between biology and the social sciences, Lumsden and Wilson (1981) extend their developments in sociobiology, using a highly mathematical formulation. In this work, they begin with the assumption that in order to understand the relationship between genetics and cultural evolution, one must examine the process of individual mental and behavioral development. The interested reader is encouraged to examine this text, as it will provide a useful and interesting insight into the mathematization of the evolution of culture. For alternate views on cultural evolution see Cavalli-Sforza and Feldman (1981) and Boorman and Levitt(1980).

Finally, many of the aforementioned references initiated further research in the field. A most recent example is Eshel and Cavalli-Sforza (1982) which discusses extensions on the concept of evolutionarily stable strategies and the evolution of cooperativeness.

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In closing this brief summary, it should be pointed out that each of the cited references has extensive literature citations. Many of these citations are worth reading. In the upcoming section we begin discussion of a simple kinetics model for the propagation of a hypothetical "idea" or "meme" in a structured population.

II. The Kinetics Model.

We begin by considering a three compartment model for a population containing pre-replicative individuals, P; replicative individuals, R; and post-replicative individuals, S; as illustrated in Fig. 1. We assign average specific mortalities μ_S, μ_P and μ_R to each compartment in units of 1/year, and average transition rate constants k_P and k_R for moving from one group into the next (aging) in units of 1/year. From Fig. 1

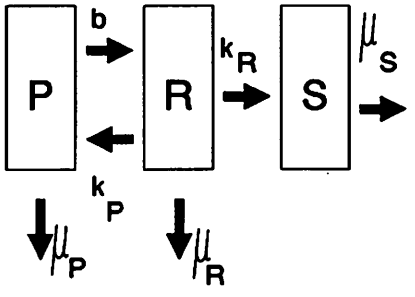

Figure 1. A simple three compartment model containing prereplicative, replicative, and postreplicative compartments. Each compartment has an average specific mortality μ_S, μ_P, μ_P, and k_P and k_R are the average rate constants for moving from one (aging) group into the next.

we deduce the following kinetics equation system

$$\frac{dP}{dt} = bR - (k_P + \mu_P)P$$

$$\frac{dR}{dt} = -bR + k_P P - (k_R + \mu_R)R \qquad (2.1a)$$

$$\frac{dS}{dt} = k_R R - \mu_S S$$

$$N = P + R + S$$

From our equation for N we obtain the additional equation

$$\frac{dN}{dt} = -(\mu_S S + \mu_R R + \mu_P P) \qquad (2.1b)$$

The problem with this model is the fact that as R feeds P, it depletes R. Hence, as $t \to \infty$ this implies everything in the system tends to zero. To rectify this problem, we need to realize that the reproductive process of R adds to P but does not deplete R, particularly in human populations. This yields the new system of equations

$$\frac{dP}{dt} = bR - (k_P + \mu_P)P$$

$$\frac{dR}{dt} = k_P P - (k_R + \mu_R)R \qquad (2.2)$$

$$\frac{dS}{dt} = k_R R - \mu_S S$$

$$\frac{dR}{dt} = bR - (\mu_S S + \mu_R R + \mu_P P)$$

System (2.2) corresponds to situation of the type illustrated in Fig. 2.

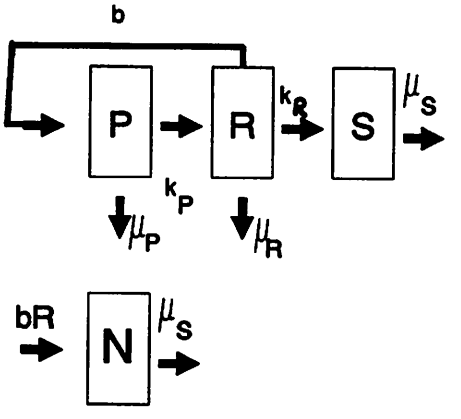

Figure 2. A simple three compartment model which accounts for the reproductive process R.

Let us now consider how this simplest class of model might be used to examine the transmission of a quantitative idea, meme, or trait. We will indicate the traited variables by the subscript T. We make it clear that we do not mean a genetic trait, when we discuss traits.

Assume we have a collection of individuals which has size $N(t)$. Let $T(t)$ be the total number of traited individuals (individuals with the "meme") in the population at time t. Then $F(t) = N(t) - T(t)$ individuals do not have the trait. If we assume a compartmentalized population of the form illustrated in Fig. 3 we may then argue as follows.

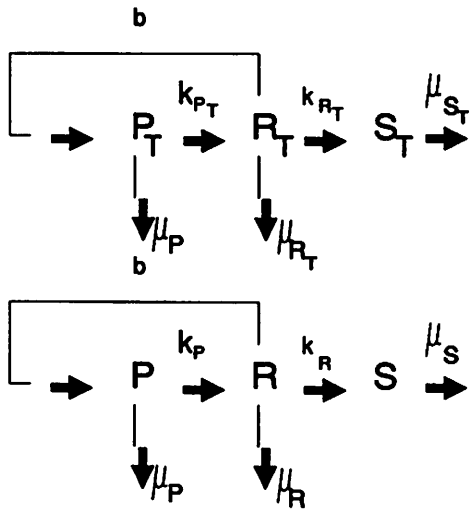

Figure 3. The uncoupled trait -no trait model for the transmission of a "culturgen" or meme.

We know that individuals in R_T may give rise to P_T and P individuals. Hence, let f_T be the fraction of P_T's produced by R_T's. Then $(1 - f_T)$ is the untraited and we obtain

$f_T b_T R_T$ = number of traited new births arising from traited individuals

$(1 - f_T) b_T R_T$ = number of nontraited new births arising from traited individuals (2.4)

Further, we will assume that R's may give rise to P_T's. Thus, we also have

fbR = number of untraited new birth arising from untraited individuals

$(1 - f)bR$ = number of traited new births arising from untraited individuals (2.4)

This is illustrated in Fig. 4.

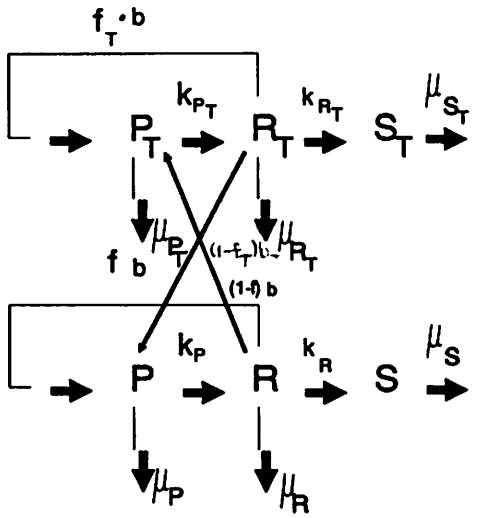

Figure 4. An illustration of the coupled trait -no trait kinetics model for the transmission of a "culturgen" or meme.

The differential equations arising from Fig. 4 are given by the following system of equations

Traited:

$$\frac{dP_T}{dt} = f_T b_T R_T + (1-f)bR - (k_{P_T} + \mu_{P_T})P_T$$

$$\frac{dR_T}{dt} = k_{P_T} P_T - (k_{R_T} + \mu_{R_T})R_T \qquad (2.5)$$

$$\frac{dS_T}{dt} = k_{R_T} R_T - \mu_{S_T} S_T$$

Untraited:

$$\frac{dP}{dt} = fbR + (1-f_T)b_T R_T - (k_P + \mu_P)P$$

$$\frac{dR}{dt} = k_P P - (k_R + \mu_R)R \qquad (2.6)$$

$$\frac{dS}{dt} = k_R R - \mu_S S$$

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Further,

$$\frac{dT}{dt} = \frac{dP_T}{dt} + \frac{dR_T}{dt} + \frac{dS_T}{dt}$$

$$= f_T b_T R_T + (1-f)bR - (\mu_{P_T} P_T + \mu_{R_T} R_T + \mu_{S_T} S_T) \tag{2.7}$$

$$\frac{dF}{dt} = fbR + (1-f_T)b_T R_T - (\mu_P P + \mu_R R + \mu_S S)$$

Notice that for $f_T = f = 1$ this system decouples completely as one would expect. Letting

$$\vec{X} = (P_T, R_T, S_T, T, P, R, S, F)^T \tag{2.8a}$$

and

$$\mathbf{A} = \begin{pmatrix} -(k_{P_T}+\mu_{P_T}) & f_T b_T & 0 & 0 & 0 & (1-f)b & 0 & 0 \\ k_{P_T} & -(k_{P_T}+\mu_{P_T}) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_{R_T} & -\mu_{S_T} & 0 & 0 & 0 & 0 & 0 \\ -\mu_{P_T} & f_T b_T - \mu_{R_T} & -\mu_{S_T} & 0 & 0 & (1-f)b & 0 & 0 \\ 0 & (1-f_T)b_T & 0 & 0 & -(\mu_P + k_P) & fb & 0 & 0 \\ 0 & 0 & 0 & 0 & k_P & -(\mu_R + k_R) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & k_R & -\mu_S & 0 \\ 0 & (1-f_T)b_T & 0 & 0 & -\mu_P & fb - \mu_R & -\mu_S & 0 \end{pmatrix}$$

equations (2.5) - (2.7) reduce to the simple matrix differential equation system

$$\frac{d\vec{X}}{dt} = \mathbf{A}\vec{X} \tag{2.9}$$

As the variable F and T are linear combinations of the other variables we may eliminate them to yield a reduced system of ode's where $\vec{Y}$ is the reduced vector

$$\vec{Y} = (P_T, R_T, S_T, P, R, S)^T \tag{2.10a}$$

and **B** is the reduced matrix

$$\mathbf{B} = \begin{pmatrix} -(k_{P_T}+\mu_{P_T}) & f_T b_T & 0 & 0 & (1-f)b & 0 \\ k_{P_T} & -(k_{P_T}+\mu_{P_T}) & 0 & 0 & 0 & 0 \\ 0 & k_{R_T} & -\mu_{S_T} & 0 & 0 & 0 \\ 0 & (1-f_T)b_T & 0 & -(\mu_P + k_P) & fb & 0 \\ 0 & 0 & 0 & k_P & (-k_R + \mu_R) & 0 \\ 0 & 0 & 0 & 0 & k_P & -\mu_S \end{pmatrix}$$

and

$$\frac{d\vec{Y}}{dt} = \mathbf{B}\vec{Y} \tag{2.11}$$

setting $det\,|\mathbf{B} - \lambda\mathbf{I}| = 0$ we obtain the characteristic equation $P(\lambda)$. That is,

$$det\,|\mathbf{B} - \lambda\mathbf{I}| = 0 = (\mu_S + \lambda)(\mu_{S_T} + \lambda) det\,|\overline{\mathbf{B}} - \lambda\mathbf{I}| = P(\lambda) = 0 \tag{2.12a}$$

where

$$\overline{\mathbf{B}} = \begin{pmatrix} -(k_{P_T}+\mu_{P_T}) & f_T b_T & 0 & (1-f)b \\ k_{P_T} & -(k_{P_T}+\mu_{P_T}) & 0 & 0 \\ 0 & (1-f_T)b_T & -(k_P+\mu_P) & fb \\ 0 & 0 & k_P & -(k_R+\mu_R) \end{pmatrix}$$

After some tedious algebra, one can show that

$$det\,|\overline{\mathbf{B}} - \lambda\mathbf{I}| = [\alpha_T(\lambda)\alpha(\lambda) - (1-f)(1-f_T)\lambda b\, b_T] \tag{2.13a}$$

Where

$$\alpha_T(\lambda) = \frac{(f_T b_T k_{P_T} - [(k_{R_T} + \mu_{R_T}) + \lambda][(k_{P_T} + \mu_{P_T}) + \mu])}{k_{P_T}} \tag{2.13b}$$

$$\alpha(\lambda) = \frac{(fbk_P - [(k_R + \mu_R) + \lambda][(k_P + \mu_P) + \lambda])}{k_P}$$

Hence, the eigenvalues are given by $\lambda = -\mu_S$; $\lambda = -\mu_{S_T}$ and the four roots of (2.13a).

For the case where either $f_T = 1$, or $f=1$, or both $f_T = f = 1$; equation (2.13a) becomes relatively straight forward to solve. See Appendix I for a discusssion of the uncoupled case, and see Appendix II for the case $f=1$, f_T arbitrary. If we let $\beta_R = (k_R + \mu_R)$, $\beta_{R_T} = (k_{R_T} + \mu_{R_T})$, $\beta_P = (k_P + \mu_P)$, and, $\beta_{P_T} = (k_{P_T} + \mu_{P_T})$ we can show that (2.13a) is a quartic equation

$$\lambda + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \tag{2.14a}$$

where

$$\begin{aligned} a_1 &= \beta_R + \beta_{R_T} + \beta_P + \beta_{P_T} \\ a_2 &= \beta_{R_T}\beta_{P_T} + \beta_R\beta_P + (\beta_R + \beta_P)(\beta_{R_T} + \beta_{P_T}) - (f_T b_T k_{P_T} + fbk_P) \\ a_3 &= (\beta_R + \beta_P)[\beta_{R_T}\beta_{P_T} - f_T b_T k_{P_T}] + (\beta_{R_T} + \beta_{P_T})[\beta_R\beta_P - fbk_P] \\ a_4 &= (f_T + f - 1)bb_T k_P k_{P_T} - \beta_R\beta_P f_T b_T k_{P_T} - \beta_{R_T}\beta_{P_T} fbk_P + \beta_R\beta_P\beta_{R_T}\beta_{P_T} \end{aligned} \tag{2.14b}$$

Ungar (1982 a;b) has discussed a fast, efficient method for the determination of the roots of (2.14a). His method will be useful in the following discussion.

From (2.12a) we have that two of the six eigenvalues of (2.11) are real and negative. Hence as $t \to \infty$ the components of the total solution to (2.11) corresponding to these eigenvalues (the eigenvectors corresponding to the eigenvalues $\lambda = -\mu_S$ and $\lambda = -\mu_{ST}$) will tend to zero. Hence, the asymptotic behavior of (2.11) is determined by the roots of the quartic (2.14). Clearly, if all the roots are real and negative we obtain an uninteresting solution; namely $\vec{Y} = \vec{O}$ is stable and all population groups die out. Should any of the real eigenvalues be positive we obtain exponential growth which is equally uninteresting. What we would like to see is persistence of "trait" or "nontrait" components such that unbounded growth does not occur. One way to force this type of dynamics is to require that at least one pair of complex eigenvalue exist, that they be pure complex, and that all four remaining eigenvalue be negative and real. Let us consider how this could occur. From this point on, we will make extensive use of results detailed in Ungar (1982b).

Let us first cite some preliminaries. Let $\lambda_0, \lambda_1, \lambda_2, \lambda_3$ be the four roots of an arbitrary quartic

$$\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0 \tag{2.15a}$$

then Ungar (1982a,b) has shown that

$$\begin{aligned} \lambda_0 &= \alpha + \beta + \gamma + \delta \\ \lambda_1 &= \alpha + \beta - \gamma - \delta \\ \lambda_2 &= \alpha - \beta + \gamma - \delta \\ \lambda_3 &= \alpha - \beta - \gamma + \delta \end{aligned} \tag{2.15b}$$

where $\alpha, \beta, \gamma, \delta$ are defined as follows. Let

$$\begin{aligned} P &= a_1^3 - 4a_1a_2 + 8a_3 \\ Q &= 12a_4 + a_2^2 - 3a_1a_3 \\ R &= 27a_1^2a_4 - 9a_1a_2a_3 + 2a_2^3 - 72a_2a_4 + 27a_3^2 \end{aligned} \tag{2.15c}$$

Further, let

$$\begin{aligned} \alpha_0 &= a_1^2 - \frac{8}{3}a_2 \\ \beta_0 &= \frac{4}{3}\sqrt[3]{\frac{R + \sqrt{R^2 - 4Q^3}}{2}} \\ \gamma_0 &= \frac{4}{3}\sqrt[3]{\frac{R - \sqrt{R^2 - 4Q^3}}{2}} \end{aligned} \tag{2.15d}$$

Then

$$\alpha = \frac{-a_1}{4} \qquad q_1 = \frac{-1 + i\sqrt{3}}{2}$$

$$\beta = \frac{1}{4}\sqrt{\alpha_0 + \beta_0 + \gamma_0} \qquad q_2 = \frac{-1 - i\sqrt{3}}{2}$$

$$\alpha = \frac{1}{4}\sqrt{\alpha_0 + q_1\beta_0 + q_2\gamma_0} \tag{2.15e}$$

$$\delta = \frac{1}{4}\sqrt{\alpha_0 + q_2\beta_0 + q_1\gamma_0}$$

Ungar (1982b) has shown that if $R^2 - 4Q^3 > 0$ that two and only two of the roots of (2.15a) are real. And, in fact α and β are pure real, and either γ and δ or $\gamma - \delta$ is pure complex and the other is pure real. Hence, if $\gamma + \delta$ is real then we must require

$$\begin{aligned} &\alpha + \beta + \gamma + \delta < 0 \\ &\alpha + \beta - \gamma - \delta < 0 \\ &\alpha - \beta = 0 \end{aligned} \tag{2.16a}$$

This insures us that, for this choice, our real eigenvalue will be negative and our complex eigenvalues will be pure complex. Hence, our four eigenvalues are

$$\begin{aligned} \lambda_0 &= 2\beta + (\gamma + \delta) \\ \lambda_1 &= 2\beta - (\gamma + \delta) \\ \lambda_2 &= \gamma - \delta \\ \lambda_3 &= -(\gamma - \delta) \end{aligned} \tag{2.16b}$$

For the case $\gamma - \delta$ real, we require

$$\begin{aligned} &\alpha + \beta = 0 \\ &\alpha - \beta + \gamma - \delta < 0 \\ &\alpha - \beta - \gamma + \delta < 0 \end{aligned} \tag{2.16c}$$

and the four eigenvalues become

$$\begin{aligned} \lambda_0 &= \gamma + \delta \\ \lambda_1 &= -(\gamma + \delta) \\ \lambda_2 &= -2\beta + (\gamma - \delta) \\ \lambda_3 &= -2\beta - (\gamma - \delta) \end{aligned} \tag{2.16d}$$

In the case $R^2 - 4Q^3 = 0$, Unger (1982b) has shown that two roots of (2.15a) are real and the remaining two roots of (2.15a) are real if $f_T \geq 0$ for all three possible cube roots in (2.17) where T is given by

$$T = 3a_1^2 - 8a_2 + 8Re\left[\sqrt[3]{\frac{R + \sqrt{R^2 - 4Q^3}}{2}}\right] \tag{2.17}$$

If we require $T < 0$, and if we define

$$T_0 = 3a_1^2 - 8a_2 - \frac{4}{3}\sqrt[3]{\frac{R}{2}} \tag{2.18}$$

then Ungar (1982b) points out that γ, δ are real for $T_0 \geq 0$. Thus, for $T_0 < 0$ α, β are real and γ, δ are pure complex (iff two values of T are negative). In this case we require

$$\begin{aligned} &\alpha+\beta=0 \\ &\alpha-\beta+\gamma-\delta<0 \\ &\alpha-\beta-\gamma+\delta<0 \end{aligned} \tag{2.19}$$

The case for $R^2-4Q^3<0$ may be analyzed in the same manner but proves to be exceedingly tedious. For this case, one can show Ungar (1982b) that at most two of the three numbers β, γ, δ may be pure complex and unequal and α is real. However, from Theorem 6.1 of Ungar (1982b) we may guarantee that all the roots are complex if $f_T < 0$ for at least one of the three possible cube roots of (2.17).

III. Closing Comments.

This concludes our discussion of the kinetics model for the distribution of a hypothetical "meme" in a population with three distinct age groups. It should be clear that this model may be extended to a multicompartment model containing more that just the three compoartments discussed in this model. A sample of a model is illustrated in Fig. 5.

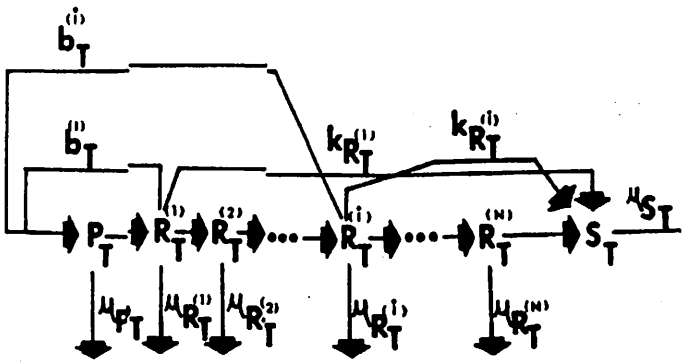

Figure 5. An illustration of a multicompartment model for the traited component.

In the next paper in this series we consider the problem from the point of view of a continuous age structure.

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Mailing Address

Department of Applications Research and Development
University of Texas System, Center for High Performance Computing
Balcones Research Center, 1.154CMS, 10100 Burnet Road, Austin, TX 78758-4497

E-mail:
M.WITTEN@CHPC.UTEXAS.EDU or M.WITTEN@UTCHPC.BITNET

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Appendix I.

In this appendix we consider the case $f = f_T = 1$. From (2.12a) and (2.13a) we obtain

$$P(\lambda) = \alpha_T(\lambda)\alpha(\lambda)(\lambda+\mu_S)(\lambda+\mu_{S_T}) \qquad (AI.1)$$

Equation (AI.1) reduces to solving the two independent equations

$$\lambda^2 + (\beta_R+\beta_P)\lambda + (\beta_R\beta_P - k_P b) = 0$$
$$\lambda^2 + (\beta_{R_T}+\beta_{P_T}) + (\beta_{R_T}\beta_{P_T} - k_{P_T} b) = 0 \qquad (AI.2)$$

One can show that the roots of (AI.2) are given by

$$\lambda_{\pm} = -\frac{(\beta_R+\beta_P) \pm \sqrt{(\beta_R+\beta_P)^2 + 4k_P b}}{2}$$
$$\lambda^T_{\pm} = -\frac{(\beta_{R_T}+\beta_{P_T}) \pm \sqrt{(\beta_{R_T}-\beta_{P_T})^2 + 4k_{P_T} b}}{2} \qquad (AI.3)$$

As $\beta_P, \beta_{P_T}, \beta_R, \beta_{R_T}, k_P, k_{P_T}$, and b are all positive, all six eigenvalues are real or have negative real parts. Hence, at best, only damped oscillation may occur.

Appendix II

In this appendix we consider $f = 1$ and f_T arbitrary. Here, (2.12a) reduces to

$$[f_T b_T k_{P_T} - (\beta_{R_T}+\lambda_{R_T})(\beta_{P_T}+\lambda)]\,[bk_P - (\beta_R+\lambda)(\beta_P+\lambda)] = 0 \qquad (AII.1)$$

For the untraited individuals the eigenvalues must satisfy

$$\lambda^2 + (\beta_R+\beta_P)\lambda + (\beta_R\beta_P - bk_P) = 0 \qquad (AII.2)$$

which yields eigenvalues

$$\lambda_{\pm} = -\frac{(\beta_P+\beta_R) \pm \sqrt{(\beta_P+\beta_R)^2 - 4(\beta_R\beta_P - k_P b)}}{2} \qquad (AII.3)$$

For the traited component of (AII.1), the eigenvalues must satisfy

$$\lambda^2 + (\beta_{R_T}+\beta_{P_T})\lambda + (\beta_{R_T}\beta_{P_T} - f_T b_T k_{P_T}) = 0 \qquad (AII.4)$$

or

$$\lambda^T_{\pm} = -\frac{\beta_{R_T}+\beta_{P_T} \pm \sqrt{(\beta_{R_T}+\beta_{P_T})^2 + 4(\beta_{R_T}\beta_{P_T} - f_T k_{P_T} b_T)}}{2} \qquad (AII.5)$$

We may write $\sqrt{(\beta_{R_T}+\beta_{P_T})^2 - 4(\beta_{R_T}\beta_{P_T} - f_T k_{P_T} b_T)}$ as

$$\sqrt{\beta^2_{R_T} + 2\beta_{R_T}\beta_{P_T} + \beta^2_{P_T} - 4\beta_{R_T}\beta_{P_T} + 4f_T k_{P_T} b_T}$$
$$= \sqrt{(\beta_{R_t} - \beta_{P_T})^2 + 4f_T k_{P_T} b_T} \geq 0 \qquad (AII.6)$$

Thus. again we have no complex eigenvalues, and hence, no oscillations.